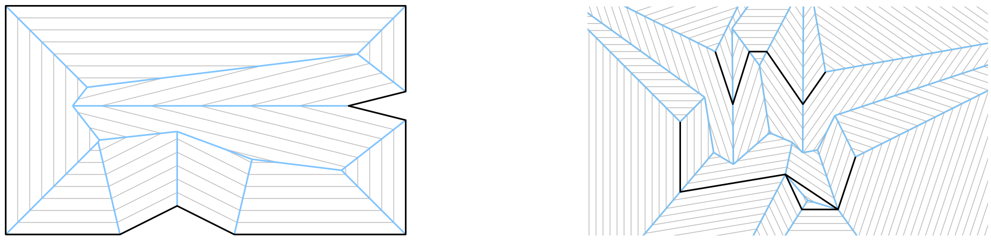
Straight Skeletons

Dissertation Disposition

Peter Palfrader — March 2014

Straight skeletons are a geometric object first introduced to computational geometry two decades ago by Aichholzer et al. [2]. Roughly speaking, the straight skeleton $\mathcal{S}(P)$ of a simple polygon P is the geometric graph whose edges are the traces of vertices of shrinking, mitered offset curves of P, see Figure 1a. Straight skeletons naturally appear in other fields such as architecture where they solve particular problems in roof design. As such, they are much older, and Peschka, for instance, discussed them in the 19th century [13].



(a) **(b)**

Figure 1: (a) The straight skeleton $\mathcal{S}(P)$ (blue) of an input polygon P (bold) is the union of traces of vertices of mitered offset curves (gray). (b) A straight skeleton of a planar straight-line graph (PSLG).

The definition of straight skeletons can be extended from polygons to planar straight-line graphs (PSLGs) [1], i.e., collections of line segments that do not intersect except at common endpoints, see Figure 1b. The *weighted straight skeleton*, a variant where not all edges of the offset move at the same speed, was first mentioned by Eppstein and Erickson [5].

In recent years, straight skeletons have become an important element in the computational geometry toolkit. Tomoeda et al. [10] use straight skeletons to create signs with an illusion of depth. Sugihara [9] uses (weighted) straight skeletons in the design of pop-up cards. Aichholzer et al. [2] and Laycock et al. [7] apply straight skeletons to roof design and terrain generation.

The currently best algorithm to construct the straight skeleton is due to Eppstein and Erickson [5] and runs in $\mathcal{O}(n^{17/11+\epsilon})$ both time and space. Unfortunately, it uses complex data structures and, thus, is unlikely to be implementable. The two fastest actual implementations have been developed here in Salzburg under the supervision of Martin Held: BONE by Stefan Huber [6] and SURFER by Peter Palfrader [8]. While the implementations achieve runtimes following an $n \log n$-law in practice, their underlying algorithms actually have worst case lower bounds in $\Omega(n^2 \log n)$.

For special cases better algorithms are known. For instance, for convex input the straight skeleton coincides with the Voronoi diagram and as such can be computed in linear time [4]. For input where the mitered offset curves never change in a particular way (no *multi-split events*, the algorithm by Vigneron and Yan [12] achieves an expected $\mathcal{O}(n^{4/3} \log n)$ time complexity.

Goals of this dissertation include

- the study of existing algorithms, such as the triangulation based algorithm by Aichholzer and Aurenhammer [1], in order to find better, maybe even tight, upper and lower bounds,
- the development and analysis of new algorithms for constructing (weighted) straight skeletons of arbitrary or special input, and
- the investigation of upper and lower bounds of the (weighted) straight skeleton problem itself, either of general or special input.

Furthermore, this dissertation will

- conduct research into applications of straight skeleton, for instance industrial applications in tool-path generation, and

- study useful variations and generalizations of straight skeletons and related structures, such as $3D$-straight skeletons [3], the linear axis [11], and motorcycle graphs [5], and their relation to one-another.

Due to the technical nature of the work, the academic degree of "Dr. Techn." is sought.

The work on this dissertation is part of Austrian Science Fund (FWF) project P25816-N15: *Theory and Application of Straight Skeletons in 2D and 3D*.

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